Nanoplasmonic planar traps - a tool for engineering p-wave interactions

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Engineering strong p-wave interactions between fermions is one of the challenges in modern quantum physics. Such interactions are responsible for a plethora of fascinating quantum phenomena such as topological quantum liquids and exotic superconductors. In this letter we propose to combine recent developments of nanoplasmonics with the progress in realizing laser-induced gauge fields. Nanoplasmonics allows for strong confinement leading to a geometric resonance in the atom-atom scattering. In combination with the laser-coupling of the atomic states, this is shown to result in the desired interaction. We illustrate how this scheme can be used for the stabilization of strongly correlated fractional quantum Hall states in ultracold fermionic gases.

Recently there has been growing interest in plasmonic nanostructures that can be used for various applications in quantum optics and atomic physics [1–7]. Particularly interesting is the possibility of confining atomic motion over regions in space of order of nanometers, comparable or smaller than typical values of the atom-atom scattering length. In such a regime, atomic scattering undergoes strong modifications due to *confinement induced resonances* [8, 9]. Here we propose to use this effect to engineer strong and robust p-wave interactions between fermionic atoms in planar geometries, which overcomes the challenges associated with creating such interactions in previously proposed techniques. This opens a new path towards the realization of exotic fractional quantum Hall states [10, 11] and superfluid phases [12, 13].

A strong motivation for realizing such states are their intriguing topological properties which find direct application in topological quantum computation, protected quantum qubits, and protected quantum memories [14]. Similarly, p-wave repulsion can stabilize low filling fractional quantum Hall states [10, 15], including the Moore-Read state [11]. This state has been proposed in the context of a pronounced fractional quantum Hall plateau at filling 5/2 [13], but formally it also resembles the spinless chiral p-wave superfluid state. In solid-state physics, only in Strontium Ruthenate, chiral $(p_x + ip_y)$-wave Cooper pairs are believed to be responsible for the observed superfluidity of electrons [16]. In the field of quantum gases, strong p-wave interaction can in principle be achieved by using Feshbach resonances. Due to the inelastic loss processes, however, a strong p-wave interaction is hard to achieve experimentally [17]. Also, in Bose-Fermi mixtures, density fluctuations of bosons can induce attractive p-wave interactions or even higher partial waves between the fermions [18–20]. However, such proposals also run into difficulties due to the phase separation instability of Bose-Fermi mixtures and stringent constraints on temperature.

In this letter we propose to combine two important concepts: strongly confined two-dimensional (2D) traps via nanoplasmonic fields, and strong laser induced synthetic gauge fields, as illustrated in Fig. 1. For simplicity, the synthetic gauge field considered is produced through a minimal scheme described in Ref. [21]. It consists of a laser field coupling two internal levels of the fermionic atoms, and an external electric or magnetic field which produces a linear variation of the energy of the internal states throughout the sample. Preparing the system in the lower dressed state, the sample is effectively subjected to a strong synthetic magnetic field [22]. Being polarized in one dressed state, interactions between the fermionic atoms are prohibited by the Pauli principle. However, the external degrees of freedom provide a small coupling to the higher dressed state. Remarkably, this will be shown to result in a residual p-wave contact interaction between the fermions. This contribution can be enhanced thanks to the resonant behavior of the atom-atom scattering length in strongly confined 2D settings [9]. This not only allows to strengthen the interaction but also to explore both attractive and repulsive p-wave interactions, i.e. going from the physics of p-wave pairing to fractional quantum Hall physics.

Experimental difficulties to provide a sufficient transverse confinement, that is on the order of the atom-atom scattering length, are surmountable thanks to the new developments in plasmonics. The interaction of cold atoms with nanoplasmonic systems has attracted significant interest recently. A notable feature of surface plasmon excitations, which exist along a metal-dielectric interface, is the lack of a diffraction limit. In the context of atom trapping, this enables the generation of fields with dramatically reduced effective wavelengths compared to free space, and a corresponding reduction of parameters such as trap confinement. The properties of the plasmons can also be greatly engineered through the underlying device geometry. The interaction between Bose-Einstein condensates and tailored plasmonic micro-potentials has recently been observed [3], and plasmon-based trapping techniques for ultracold atoms with applications in quantum simulation have been proposed [1, 2, 4].

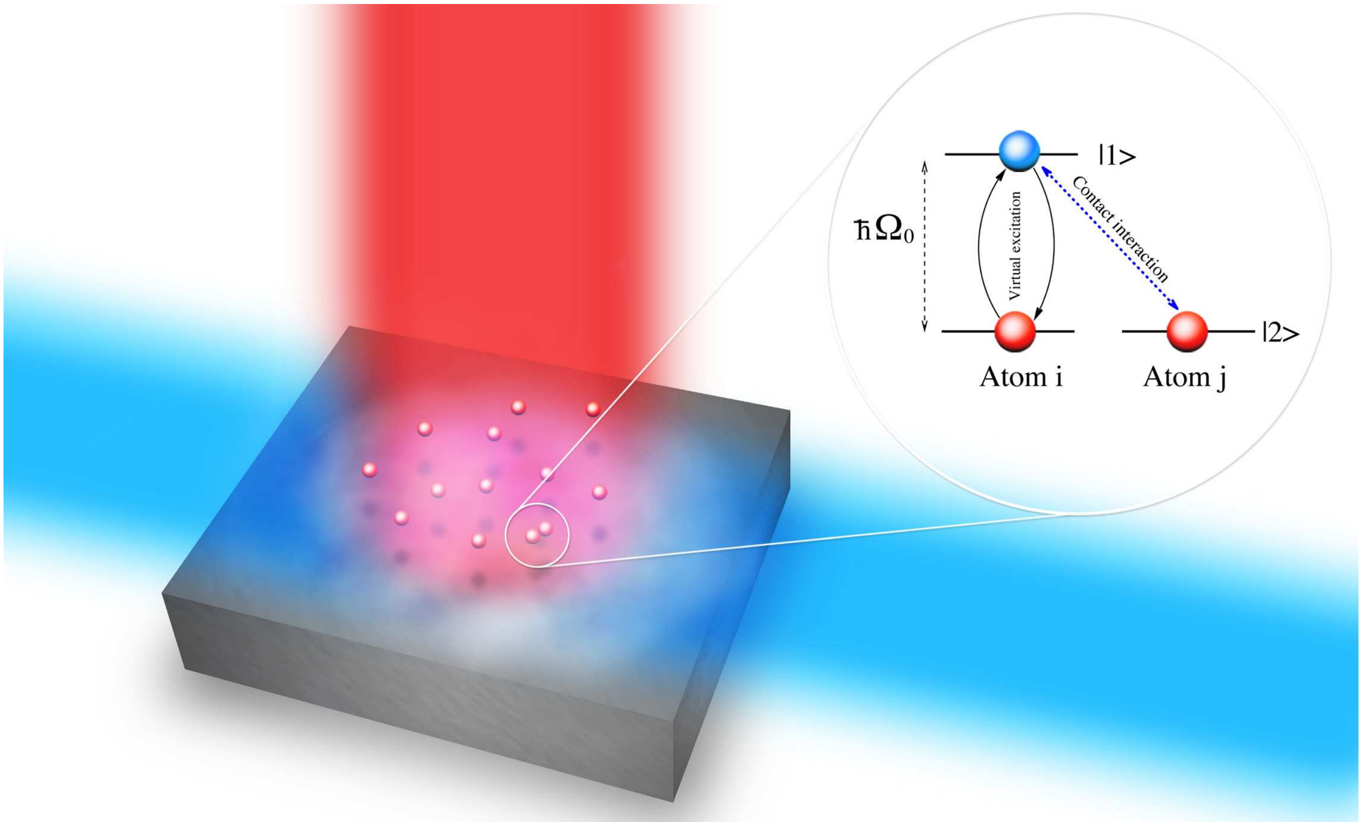

Figure 1. The ultracold atomic sample is tightly confined above a metallic surface by a nanoplasmonic field produced by an external laser (in red) pointing perpendicular to the metal surface. A second laser (in blue), shining in from the side, is used to generate the artificial magnetic field felt by the atoms. The mechanism for fermion-fermion contact interaction involves the virtual excitation of one of the atoms into the excited dressed state, as illustrated in the inset.

As a straightforward application of the scheme, we will consider the case of repulsive interactions and show by exact diagonalization how the induced p-wave interaction can be used to explore different quantum Hall phases, notably going from filled Landau level (LL) physics, to the fractional quantum Hall regime with a $\nu = 1/3$ Laughlin state [10], passing through a phase with sizable overlap with the Pfaffian state [11].

Model

We consider a trapped ultracold gas of fermionic atoms with two internal states $|\mathrm{g}\rangle, |\mathrm{e}\rangle$. The single particle Hamiltonian $H_{\mathrm{sp}} = H_{\mathrm{ext}} + H_{\mathrm{AL}}$ consists of an external part $H_{\mathrm{ext}} = p^2/(2M) + V(\mathbf{r})$ with the anisotropic trapping potential $V(\mathbf{r})$, and an atom-laser coupling H_{AL} including also the internal energies. This coupling is responsible for a synthetic gauge field which emerges due to the accumulation of Berry's geometrical phase when an atom moves within the laser field [23]. The key to achieving non-vanishing phases on closed contours is to make the internal energies, and thus H_{AL}, spatially dependent via a Stark or Zeeman shift, such that H_{AL} and H_{ext} do not commute (see Methods).

The laser light mixes the ground and excited state, giving rise to position-dependent *dressed states*, $|\Psi_1\rangle$ and $|\Psi_2\rangle$, which are the eigenstates of the atom-laser interaction. As detailed in the Methods section, increasing the laser strength, and thus the Rabi coupling between the two bare states, one can energetically favor one dressed manifold, say $|\Psi_2\rangle$. By adjusting the external trapping, the single particle Hamiltonian projected in this lowest dressed manifold can be written as the usual quantum Hall one,

$$H_{22} = \frac{(\boldsymbol{p} + \boldsymbol{A})^2}{2M} + \frac{M\omega_\perp^2}{2}(1-\eta^2)(x^2+y^2) \qquad (1)$$

where $\omega_\perp$ is the effective xy trapping frequency, $\boldsymbol{A} = \hbar\eta(y,-x)/\lambda_\perp^2$, and η is the strength of the synthetic gauge field which depends on the laser wavenumber k and the spatial extent of the Stark or Zeeman shift w (see Methods). This Hamiltonian has the well-known LL structure, and its eigenfunctions are the Fock-Darwin states. Restricting ourselves to the lowest LL, the corresponding wave functions read $\varphi_l^{\mathrm{FD}}(z) \propto z^l \exp(-|z|^2/\lambda_\perp^2)$ where $z = x - iy$ describes the atom position in the (x, y) plane, and $\lambda_\perp = \sqrt{\hbar/(M\omega_\perp)}$. The adiabatic approxi-

mation requires large Rabi frequencies $\hbar\Omega_0 \gg E_R$ [22], where the recoil energy is $E_R = (k^2\lambda_\perp^2/2)(\hbar\omega_\perp)$. Within this limit, the off-diagonal Hamiltonian elements, H_{12} and H_{21}, connecting the dressed states are neglected. Then, transitions to the higher dressed manifold are fully suppressed. A general atomic state, $\chi(\boldsymbol{r}) = \tilde{\varphi}_1(\boldsymbol{r}) \otimes |\Psi_1\rangle + \tilde{\varphi}_2(\boldsymbol{r}) \otimes |\Psi_2\rangle$, becomes a low-lying solution for $\tilde{\varphi}_1 = 0$ and $\tilde{\varphi}_2 = \varphi_l^{\mathrm{FD}}$. In our approach, however, some amount of non-adiabaticity is crucial, as it will yield a finite value for $\tilde{\varphi}_1$ resulting in non-zero contact interactions.

p-wave fermion-fermion interaction

Now we turn to the atom-atom interactions, which we take as contact interactions. In terms of the bare fermionic states, it reads

$$V_{ij} = g_c \frac{\hbar^2}{M}\, \delta(z_i - z_j)(|\mathrm{e}\rangle\,|\mathrm{g}\rangle\,\langle\mathrm{e}|\,\langle\mathrm{g}| + |\mathrm{g}\rangle\,|\mathrm{e}\rangle\,\langle\mathrm{g}|\,\langle\mathrm{e}|)\,. \quad (2)$$

Here, g_c is a number quantifying the interaction strength. A more precise definition will be given later. Of course, in the dressed basis the interaction term maintains its form, such that interactions remain restricted to pairs of one atom in $|\Psi_1\rangle$ and the other in $|\Psi_2\rangle$. Thus, by polarizing the system in the lower dressed state $|\Psi_2\rangle$, no interactions are present in the adiabatic limit $\Omega_0 \to \infty$. Still, by making the ratio of the Rabi frequency to recoil energy much bigger than 1, $R_E \equiv \hbar\Omega_0/E_R \gg 1$, we can work in a quasi-polarized regime, in which the $|\Psi_1\rangle$ level serves only as a virtual manifold.

In this limit, the unperturbed many-body Hamiltonian is given by

$$H^{(0)} = \sum_{i=1}^{N} H_{22}^i \mathcal{P}_i\,. \quad (3)$$

where the operator $\mathcal{P}_i = |\Psi_2\rangle_i \langle\Psi_2|_i$ projects the ith particle onto the low-lying Hilbert space. The off-diagonal terms, $H_{12}\,|\Psi_1\rangle\,\langle\Psi_2|$ and $H_{21}\,|\Psi_2\rangle\,\langle\Psi_1|$, and the atom-atom interaction of Eq. (2) are taken as perturbations. They give second-order corrections. The effective many-body Hamiltonian can then be written as

$$\begin{aligned} H^{\mathrm{eff}} &= H^{(0)} + H^{(1)} + H^{(2)} \quad \text{with} \\ H^{(1)} &= -\sum_i \frac{H_{21}^i\ H_{12}^i}{\hbar\Omega_0}\mathcal{P}_i \\ H^{(2)} &= \sum_{ij} \mathcal{P}_i \frac{H_{21}^i\ V_{ij}\ H_{12}^j}{(\hbar\Omega_0)^2}\mathcal{P}_j\,. \end{aligned} \quad (4)$$

Note that the denominator in $H^{(1)}$ has been set to a constant equaling the energy difference between dressed states $|\Psi_2\rangle$ and $|\Psi_1\rangle$. As this is taken to be large, it is the dominant contribution to the energy gap.

In a previous study of a bosonic system [22, 24], we have analyzed the influence of $H^{(1)}$, but the many-body contribution $H^{(2)}$ has been negligible due to the bosonic nature of the atoms. We will in the following show that in the fermionic case, where $H^{(2)}$ is the only many-body contribution, it becomes crucial. As illustrated in Fig. 1, $H^{(2)}$ describes a process where one atom is excited from $|\Psi_2\rangle$ to the virtual $|\Psi_1\rangle$ manifold, where it interacts with an atom in $|\Psi_2\rangle$, to then get de-excited to $|\Psi_2\rangle$ again. Importantly, acting among fermions, the many-body interaction term gives solely non-zero p-wave contributions. This is seen by using Eq. (13) to cast $H^{(2)}$ into p-wave form (cf. Ref. [15]),

$$H^{(2)} \propto \sum_{i,j} \hat{p}_{ij}\ \delta^{(2)}(z_{ij})\hat{p}_{ij}\ \mathcal{P}_i\mathcal{P}_j \quad (5)$$

with the relative variables, $\hat{p}_{ij} = -i\hbar(\partial_{z_i} - \partial_{z_j})$ and $z_{ij} = z_i - z_j$. The important feature of Eq. (4) is that the effective interaction is linear in the bare one V_{ij}, which allows one to change the interaction from attractive to repulsive. This is in contrast to second order mechanims like the Kohn-Luttinger [25].

The main question which arises at this point is whether the residual interaction term, Eq. (5), is strong enough to significantly modify the physics of the system. This becomes possible by tuning the interaction strength g_c. It is well known that this parameter crucially depends on the geometry of the system. In particular, for transversal confinements on the order of the scattering length, and considering the case of attractive interaction the effective 2D coupling is known to behave as [9]

$$g_c = \frac{4\pi\hbar^2}{M} \frac{1}{\sqrt{2\pi}\lambda_z/a_{3D} + \log(0.918\hbar\omega_z/\pi\epsilon)}\,, \quad (6)$$

where ϵ is the energy of the motion in the $x-y$ plane and a_{3D} the 3D scattering length. For a value of $\hbar\omega_z/\epsilon = 10^3$, it produces a resonant behavior for values of the transverse confinement $\lambda_z \sim 0.4|a_{3D}|$. This confinement-induced resonance behaviour is not present in usual experiments with optical traps. There, the trapping on the z direction has at most been of the order of hundreds of nanometers, far from the resonance region. The transverse confinement lengths of $\lambda_z \sim$ 5-10 nm needed to facilitate significant interactions can be achieved using novel plasmon-based trapping techniques, such as those investigated theoretically and experimentally in Refs. [1–4]. For example, it is possible to tailor a two-dimensional array of metallic nanosystems (such as nanoshells [4]), which creates a near-planar trapping potential arising from spatial interference between an incident field and plasmon-enhanced near-field. The effective wavelength characterizing this trapping potential scales like the characteristic size of an individual nanosystem, $\lambda_{\mathrm{eff}} \sim r$, even for system sizes far below the free-space wavelength $r \ll \lambda_0$. This yields a corresponding reduction of $\sim \sqrt{r/\lambda_0}$ in the trap spatial confinement compared to free-space techniques.

This resonant behavior can in principle be used to produce arbitrarily large values of g_c and, importantly, allows to achieve not only large values of the coupling, but

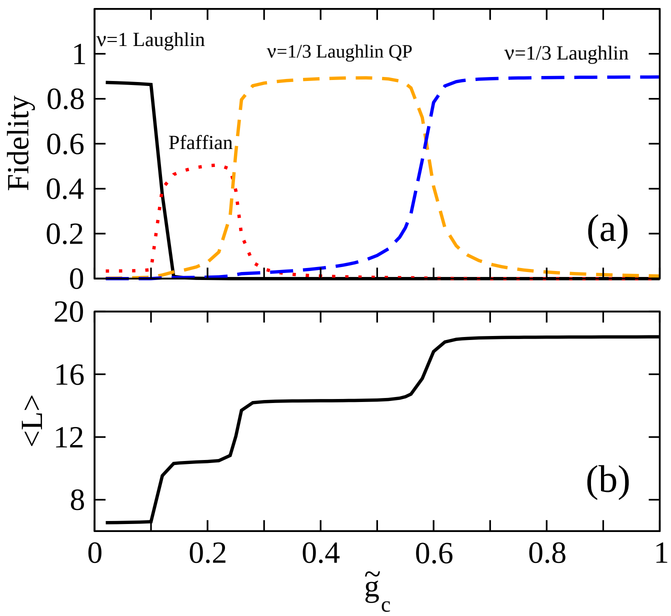

Figure 2. Evolution of the ground state as a function of the interaction parameter $\tilde{g}_c = g_c/(k\lambda_\perp\ \eta R_E)^2$. Panel (a) presents the overlap of the ground state with the filled LL state (solid-black), the fermionic Pfaffian state (dotted-red), the quasi-particle state over the $\nu = 1/3$ Laughlin (dashed-orange) and the $\nu = 1/3$ Laughlin (long-dashed-blue). Panel (b) contains the average angular momentum of the ground state of the system.

also provides a way of producing both attractive and repulsive p-wave interactions between the fermions.

Example: stabilization of the $\nu = 1/3$ Laughlin state

As an example, we discuss the case of repulsive p-wave interaction. In the context of quantum Hall physics we have to ask whether the obtained p-wave interaction is capable of bringing the system from the integer quantum Hall regime of a non-interacting system to the fractional quantum Hall regime. In that case, a Laughlin-like state should show up as the ground state of the system [1]. Note that in the quantum Hall regime, $\eta \to 1$, the contribution of H_{22} reduces to a constant, as all Fock-Darwin states become (quasi)degenerate. Thus, to bring the system into the fractional quantum Hall regime, the interaction term must be comparable to the contribution of $H^{(1)}$ term, which breaks the rotational symmetry [24].

To give definite numerical predictions, we perform an exact diagonalization (see Methods) with a few number of atoms, $N = 4$. The parameters of the system are taken as $k = 10/\lambda_\perp$, $\hbar\Omega_0 = 100E_R$ and $\eta = 0.98$. We discuss the different phases appearing as we vary the interaction strength, g_c. For weak interactions, the ground state of the system has a large overlap with the analytical form of the filled LL state, $\nu = 1$, as depicted in Fig. 2. The angular momentum of the ground state is found to be slightly larger than the analytical value, $L = 6$. As explained in Refs. [22, 24], this is due to the derivation from rotational symmetry. When the interaction is increased, the system undergoes a transition into a phase, where the ground state has large overlaps with the fermionic Moore-Read state [11]. Even stronger interactions bring the system into a state which resembles the quasiparticle excitation of the $\nu = 1/3$ Laughlin state. At another critical value of g_c, one finally reaches the $\nu = 1/3$ Laughlin state. As in the case of bosons with contact interactions, this state has zero interaction energy, and thus for any stronger interaction parameter, it remains the ground state.

Summary

We have presented a novel mechanism to realize sizable p-wave interactions between fermionic atoms. The key is the combination of a strongly confining plasmonic field, which allows to explore confinement-induced resonances, with a simple scheme to generate a strong synthetic gauge field. To exemplify the potential of our approach, we have considered the case of repulsive p-wave interaction. We have shown that our proposal allows to stabilize a number of interesting quantum Hall states, like the Pfaffian, and the $\nu = 1/3$ Laughlin state. In our numerical calculation we have considered a small number of atoms, as has become experimentally feasible recently [26–28], but we note that the scheme should also be applicable to large systems. A good candidate for realizing the proposal are Ytterbium atoms due to the long-lived state of the clock transition. Requiring an ultratight trapping in a 2D geometry, our proposal shall trigger the use of nanoplasmonic fields as a promising technique for achieving that goal.

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[1] Let us recall the Laughlin wave function [10] at filling ν,

$$\Psi_{\text{Laughlin}} = \mathcal{N} \prod_{i<j} (z_i - z_j)^{1/\nu} \mathrm{e}^{-|z|^2/2} . \tag{7}$$

which has $L_z = \nu^{-1} N(N-1)/2$.

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METHODS

Single particle Hamiltonian

The single particle Hamiltonian reads,

$$H_{\text{sp}} = H_{\text{ext}} + H_{\text{AL}} \tag{8}$$

where $H_{\text{ext}} = p^2/(2M) + V(\mathbf{r})$ with the anisotropic trapping potential $V(\mathbf{r})$. H_{AL} is the atom-laser coupling, which includes the internal energies. To make H_{AL} spatially dependent, such that H_{AL} and H_{ext} do not commute, we perform a Stark or Zeeman shift of the internal energies. The strength of this shift can be characterized by a length scale w, which is chosen such that the energies of the bare internal states read $E_g = -\hbar\Omega_0 x/(2w)$ and $E_e = \hbar\omega_A + \hbar\Omega_0 x/(2w)$. Here, ω_A is the energy difference of the bare states. In this way, preparing the system in the ground-state of H_{AL}, the external part will stimulate transitions into the excited manifold of H_{AL}. The probability of such transitions is controlled by the Rabi frequency Ω_0 of the coupling. The laser frequency is set to resonance with the atomic transition. Furthermore, we choose the laser to be a running wave in y-direction with wavenumber k. Then, within the rotating-wave approximation, the atom-laser Hamiltonian H_{AL} can be written in terms of bare states $|\mathrm{e}\rangle$ and $|\mathrm{g}\rangle$ as [29]

$$\begin{aligned}\hat{H}_{\text{AL}} = \frac{\hbar\Omega}{2} &\left[\cos\theta\left(|\mathrm{e}\rangle\langle\mathrm{e}| - |\mathrm{g}\rangle\langle\mathrm{g}|\right)\right. \\ &\left.+ \sin\theta\left(e^{i\phi}|\mathrm{e}\rangle\langle\mathrm{g}| + \text{h.c.}\right)\right],\end{aligned} \tag{9}$$

where $\Omega = \Omega_0\sqrt{1 + x^2/w^2}$, $\tan\theta = w/x$, and $\phi = ky$. Note that spontaneous emission processes are not considered in the Hamiltonian of Eq. (9). This is justified if the two atomic states are sufficiently long-lived, as is the case for the ${}^1S_0 \to {}^1P_1$ clock transition in Ytterbium. In contrast to the bosonic Ytterbium isotopes, the finite spin of the fermionic isotopes yields a small magnetic moment, which allows for a strong coupling of the clock states at reasonable laser power. Thus, achieving large Rabi frequencies, as required by our proposal, poses no problem for ^{171,173}Yb [30].

Diagonalizing Eq. (9) yields the dressed states, $|\Psi_1\rangle = \mathrm{e}^{-iG}\left(C\,\mathrm{e}^{i\phi/2}|\mathrm{g}\rangle + S\,\mathrm{e}^{-i\phi/2}|\mathrm{e}\rangle\right)$, $|\Psi_2\rangle = \mathrm{e}^{iG}\left(-S\,\mathrm{e}^{i\phi/2}|\mathrm{g}\rangle + C\,\mathrm{e}^{-i\phi/2}|\mathrm{e}\rangle\right)$, where $C = \cos\theta/2$, $S = \sin\theta/2$, $G = \frac{kxy}{4w}$. The single-particle Hamiltonian H_{sp} can be expressed as a 2×2 matrix H_{ij}. In the dressed state basis, its diagonal terms can be written as [22],

$$H_{jj} = \frac{(\boldsymbol{p} - \epsilon_j\boldsymbol{A})^2}{2M} + U + V + \epsilon_j\frac{\hbar\Omega}{2}, \tag{10}$$

with $\epsilon_1 = 1$ and $\epsilon_2 = -1$. Full expressions for the vector potential $\boldsymbol{A}$ and the scalar potential U are given in Ref. [22]. Note that for $w \ll x, y$, we recover the symmetric gauge expression $\boldsymbol{A}(\boldsymbol{r}) = \frac{\hbar k}{4w}(y, -x)$. With a convenient choice of the trapping potential, H_{22} can be made symmetric. Then, the Hamiltonian element H_{22} reads

$$H_{22} = \frac{(\boldsymbol{p} + \boldsymbol{A})^2}{2M} + \frac{M\omega_\perp^2}{2}(1 - \eta^2)r^2 \tag{11}$$

where $\omega_\perp$ is the effective xy trapping frequency, $\eta = (k\lambda_\perp^2)/(4w)$, and $\lambda_\perp = \sqrt{\hbar/(M\omega_\perp)}$. The recoil energy of the atoms is defined as $E_R = (k^2\lambda_\perp^2/2)(\hbar\omega_\perp)$.

Retaining up to quadratic terms, the off-diagonal Hamiltonian elements, $H_{12} = H_{21}^\dagger$, explicitly read

$$\begin{aligned}H_{12} &\simeq -\frac{\hbar^2}{2M}\left[-ik\partial_y\Psi + \left(\frac{k^2x}{4w} + \frac{iky}{4w^2}\right)\Psi + \frac{1}{w}\partial_x\Psi\right] \\ &= -\frac{\hbar^2}{4M}\left[\hat{a}c_1 + \hat{a}^\dagger c_2 + \hat{b}c_3 + \hat{b}^\dagger c_4\right],\end{aligned} \tag{12}$$

with $\hat{a}^\dagger \equiv -\lambda_\perp\partial_{\bar{z}} + \lambda_\perp^{-1}\frac{1}{2}z$, $\hat{a} \equiv \lambda_\perp\partial_z + \lambda_\perp^{-1}\frac{1}{2}\bar{z}$, $\hat{b}^\dagger \equiv -\lambda_\perp\partial_z + \lambda_\perp^{-1}\frac{1}{2}\bar{z}$, and $\hat{b} \equiv \lambda_\perp\partial_{\bar{z}} + \lambda_\perp^{-1}\frac{1}{2}z$. Acting on a Fock-Darwin state the operators $\hat{a}$ ($\hat{a}^\dagger$) decrease (increase) the l quantum number by one, while the operators $\hat{b}$ and $\hat{b}^\dagger$ change the Landau level.

As we will be interested in the fractional quantum Hall regime of large synthetic magnetic field, $\eta \simeq 1$, it is possible to safely neglect the $\hat{b}$ and $\hat{b}^\dagger$ contributions. In this limit, we have $c_1 = c_2 \simeq 8w/\lambda_\perp^3$, and $c_3 = -c_4 \simeq 2/(w\lambda_\perp)$. In our numerics, we will furthermore choose $w \simeq 2.5\lambda_\perp$ and $k = 10/\lambda_\perp$, implying $\eta \simeq 1$, guaranteeing $c_1 \ll c_3$. We can then write

$$H_{12} = -\frac{2w\hbar^2}{M\lambda_\perp^3}(\hat{a} + \hat{a}^\dagger) + \mathcal{O}[(w/\lambda_\perp)^{-2}]. \tag{13}$$

Exact diagonalization

To solve the effective Hamiltonian, we perform exact diagonalization. Therefore, we build many-body states using as single particle states the Fock-Darwin states, $|l\rangle$. Then the second quantized form of $H^{(2)}$ is

$$H^{(2)} = \frac{1}{2}\sum_{ij,kl}\hat{f}_i^\dagger\hat{f}_j^\dagger\hat{f}_k\hat{f}_l\ V_{ij,kl}, \tag{14}$$

where $\hat{f}_i$ anihilates an atom in $\varphi_i^{\text{FD}}(z)$. The matrix element reads, $V_{ij,kl} = (\hbar\Omega_0)^{-2}\langle i|\langle j|H_{21}VH_{12}|l\rangle|k\rangle$. Taking into account the Pauli principle, we get,

$$\begin{aligned}V_{ij,kl}^{(2)} = g_c\,\hbar\omega_\perp\Big[&(\varphi_i^*h_j^*\varphi_kh_l) - (\varphi_i^*h_j^*h_k\varphi_l) \\ &-(h_i^*\varphi_j^*\varphi_kh_l) + (h_i^*\varphi_j^*h_k\varphi_l)\Big],\end{aligned} \tag{15}$$

where $h_l \equiv [H_{12}/(\hbar\Omega_0)]\ \varphi_l^{\text{FD}}$. With the expression for H_{12} from Eq. (13), h_l is directly found to be $h_l = (k\lambda_\perp\ \eta R_E)^{-1}(\sqrt{l+1}\ \varphi_{l+1} + \sqrt{l}\ \varphi_{l-1})$. It is worth noting that due to the contact nature of the interaction, Eq. (14) commutes with $\hat{L}_z$.

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